Abelian surfaces over finite fields containing no curves of small genus

Elena Berardini
CNRS; IMB, Université de Bordeaux

In [1] it is shown that algebraic geometry codes constructed from abelian surfaces defined over a finite field respect a better bound on their minimum distance when the surface does not contain curves of small genus. This leads to the question of characterising such abelian surfaces. For genus up to 2 the problem is solved in [1], where the authors describe all Weil polynomials of abelian surfaces which do not contain irreducible curves of genus 0, 1, nor 2.

In this talk we will discuss the question of which abelian surfaces do not contain genus 3 curves, among those not containing curves of genus less than or equal to 2. In contrast with the situation dealt with in [1], the result does not only depend on the isogeny class. Firstly, we will show that, for an abelian surface without curves of genus up to 2, containing a curve of genus 3 is equivalent to admit a polarization of degree 4. This leads us to use tools developed by Howe in a series of papers [2, 3], characterising when a surface in a isogeny class has a prescribed polarization. With this theory at hand, we will be able to characterise isogeny classes with at least one abelian surface containing a genus 3 curve, or, equivalently, isogeny classes not containing curves of genus less or equal than 3. Finally, we will also comment on some properties of genus 3 curves lying on an abelian surface which do not contain irreducible curves of genus 0, 1, nor 2.

The talk is based on a work in progress with A. J. Giangreco–Maidana and S. Marseglia.

References

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