Isoperimetric inequalities for Steklov-Laplacian eigenvalues

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In a celebrated paper [3], Weinstock proves that the first non-zero Steklov Laplacian eigenvalue, among planar simply connected sets of given perimeter, is maximized by the disk. The resulting functional inequality was named after him and belongs nowadays to the folklore of applied mathematics. More recently Brock [2] showed that, if the perimeter constraint is replaced by a measure constraint, no topological restriction is needed, and moreover the inequality holds true in any dimension. In a recent paper [1] we show how to generalize the original Weinstock inequality in any dimension, in the class of convex sets with prescribed surface area. We use the inverse mean curvature flow to together with shape derivative arguments. The key result is the proof of a sharp isoperimetric inequality involving simultaneously the surface area, the volume and the boundary momentum of convex sets.

References

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