NAP 2019, CLASS #2, MAY 08, 2019

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• A little more on right cosets, First Iso Thm, etc. Defined the product AB, where A and B are *subsets* of a group G and pointed out that when $N \lhd G$ the product $(H_{g_1})(H_{g_2})$ is just Hg_1g_2.

• Discussion of permutation groups. We seem to be using S_n instead of the book's Σ_n. Sorry, Book! Defined "even" and "odd" permutation in the usual way (parity in number of transpositions used to express the permutation as a product of transpositions. Stated, without proof (because proofs tend to be a bit unpleasant) that no permutation is both even and odd. Stated that the sign of σ is the parity of the number of "switches" or "reversals", that is the number of pairs (i, j) with $i < j$ but $\sigma(i) > \sigma(j)$. This is not a useful definition, but it does give a well-defined notion. It's not hard to see that this definition works by showing that composition with an adjacent transposition either adds 1 or subtracts 1 from the number of switches (and using the fact, easily proved, that every permutation is a product of adjacent transpositions). Pointed out that A_n is the kernel of the sign map from S_n to the mulitplicative group $\{\pm 1\}$ and hence has index two. Showed that a k-cycle is the product of $k - 1$ transpositions, and hence is even if and only if k is odd.

• Jabbered a bit about subgroups of S_4. Showed that there are 9 subgroups of order 2. Identified the 4 subgroups of order 6 (copies of S_3) and noted that none is contained in A_4 thereby showing that the "converse" of Lagrange's Theorem can fail. I stated, without proof, that S_4 has 30 subgroups. Students are encouraged to play with S_4. For example, find subgroups H_1, H_2, H_3 such that $H_1 \lhd H_2$ and $H_2 \lhd H_3$ but H_1 is not normal in H_3.

• Defined "field" and gave examples: $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$, and $\mathbb{Q}(\sqrt{3})$.

• Started jabbering about vector spaces.